Some Remarks on the Final State Interactions in $B \to \pi K$ Decays

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Abstract

Careful discussions are made on some points which are met in studying B decay final state interactions, taking the $B^0 \to \pi^+ K^-$ process as an example. We point out that π–exchange rescatterings are not important, whereas for D^* and D^{**} exchanges, since the $B^0 \to D^+ D_s^-$ decay has a large branching ratio their contributions may be large enough to enhance the $B \to \pi K$ branching ratio significantly. However our estimates fail to predict a large enhancement.

The importance of studying final state interactions (FSIs) in the system of the B meson hadronic decay products is well known as the FSI is of strong interaction nature and contributes the main uncertainty in extracting the CKM matrix elements and the information of direct CP violations from B decay experiments. Even though the studying of FSI effects is very difficult it has drawn increasing attentions in recent years. Despite of some controversies exist in the literature we believe some qualitative and/or semi-quantitative results can be obtained [1,2]. The method is based on a Regge pole model description to the partial–wave rescattering amplitude. The contribution from absorption effects to the low partial–wave amplitudes is also estimated and it was pointed out that, for the meson–meson scatterings via Reggeon exchanges the absorption effects remain small [1], and absorption effects further reduce the (low partial–wave) final state rescattering effects. The theoretical tool suitable for studying the FSI effects are the Watson–Migdal theorem for final state interactions and the multi-channel N/D method [1,2]. The main uncertainty come from inelasticity is expected to be small as the inelastic contribution to a given exclusive $2 \to 2$ rescattering is of non-leading order and is expected to cancel each other and the net effect remains small [1,2]. Under these considerations meaningful numerical results are obtainable and it was found that for a quantity controlled by Regge pole exchanges, the FSI effect remains small.

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For example, for the charge–exchange rescattering the typical enhancement factor is of order $O(\lambda^2)$ [2] where λ is the Wolfenstein parameter. The strong interaction phase difference between two isospin amplitudes is also controlled by Regge pole exchanges and be a small quantity. These results are found to be consistent with experiments [3,4] involving a D meson in the final decay products.

The results mentioned above implies that a calculation based on the approximation by neglecting the FSI effects can be a reasonably good approximation in many situations unless in the case when bare amplitudes which can switch to each other via final state rescatterings differ by a large amount. In such a case the tiny FSI effects are compensated by the huge decay amplitude from which the intermediate rescattering particles are generated. Such a process may generate sizable CP violation effects [5], therefore a careful numerical study on the rescattering effects is very interesting physically. In this note we investigate some subtle points which appear in the calculation of FSI effects. The first is the rescattering process dominated by Reggeized pion exchange, which was not discussed as much as the spin–1 exchange processes and controversial results exist in the literature. We think it is worthwhile to give a careful analysis on such processes. An example for rescattering process via pion exchange is, $B^0 \to K^{*+}\rho^- \to K^+\pi^-$. the pion has a tiny mass that the pole at $t = 0.02GeV^2$ is appreciately felt in the scattering region of $t < 0$, therefore the pion exchange contribution can be important. The $B \to K\pi$ process is recently measured by experiments [7] which stimulates many theoretical discussions[8]. We use Eq. (14) derived in Ref. [2] for our numerical studies, which reads,

$$\mathbf{A}_{i\to j} = \mathbf{A}_i\{\frac{\mathrm{P}}{\pi}\int \frac{\rho(s')\mathbf{T_{ij}}(s')}{s'-s}ds' + i\rho\mathbf{T}_{ij}\}\ , \tag{1}$$

where $\mathbf{A}_{i\to j}$ is a weak decay ($B \to j$) amplitude mediated by a non-diffractive final state rescattering ($i \to j$) and $\mathbf{A}_i$ is the decay amplitude renormalized by diffractive rescatterings and may be identified as the observed physical amplitude as long as it is not too small in the class of decay amplitudes which can switch to each other via (non-diffractive) final state interactions. The partial wave rescattering amplitude is $T_{ij} = T_{K^{*+}\rho^-\to K^+\pi^-}$ in the present case.

For the simple helicity non-flip amplitudes it is argued that the absorption effects remain small for meson–meson rescatterings. [1] However the pion exchange processes of helicity–flip amplitudes is well described phenomenologically by the Williams model [6] in which the absorption effects is taken into account explicitly. Without such a careful treatment to the absorption effects one can be led to misleading conclusion as will be shown below.

For a given (s–channel) helicity, the pion exchange amplitude is [6],

$$T^{\lambda_3\lambda_1}_{\lambda_4\lambda_2} = -(-t'/4M^2)^{n/2}(-m_\pi^2/4M^2)^{x/2}\beta_e^{\lambda_3\lambda_1}\beta^e_{\lambda_4\lambda_2}R[s,\alpha_e(t)] \ , \qquad (2)$$

where $R[s,\alpha_e(t)]$ is the Reggeized propagator,

$$R[s,\alpha_e(t)] = \frac{1}{2}[1+(-)^{s_e}e^{-i\pi\alpha_e(t)}]\Gamma[l_e-\alpha_e(t)](\alpha')^{1-l_e}(\alpha' s)^{\alpha_e(t)} \ , \qquad (3)$$

and $l_e = s_e = 0$ and $\alpha_e(t) = \alpha'(t-m_\pi^2)$ for the pion exchange ($\alpha' = 0.93GeV^{-2}$ is the universal slope parameter for light hadrons). In equation (2) $t' = t - t_{min}$, M is the nucleon mass, $n = |\lambda_3 - \lambda_1 - \lambda_4 + \lambda_2|$ is the net helicity flip and $n+x = |\lambda_3-\lambda_1|+|\lambda_4-\lambda_2|$. In equation (2) absorption effects have been taken into account to explain the forward spike observed in experiments [6]. Had we omitted the absorption effects the term $(-m_\pi^2/4M^2)^{x/2}$ in equation (2) would be replaced by $(-t'/4M^2)^{x/2}$. Roughly speaking, the inclusion of absorption effects is equivalent to subtracting the s–wave component from the full T matrix element. In our present case of $B \to {K^*}^+\rho^- \to K^+\pi^-$ there are two helicity amplitudes, T_{11} (=T_{-1-1}) and T_{00}, with $n=0, x=2$ and $n=x=0$, respectively. We use the Reggeon coupling constants from that of Ref. [6], $\beta_\pi^{\lambda_\rho=1}(\rho^0\pi^-) = 3.45$ and $\beta_\pi^{\lambda_\rho=0}(\rho^0\pi^-) = 4.40$, respectively. Other coupling constants are estimated by SU(3) relations. Taking the s–wave projection of the T matrix element and making use of Eq.(1) we get,

$$R^{00} \equiv \frac{A^{00}(B\to K^{*+}\rho^- \to K^+\pi^-)}{A^{00}(B\to K^{*+}\rho^-)} = \ (0.24+0.71i)\times 10^{-2} \ . \qquad (4)$$

The absorption effects severely reduce the s–wave rescattering amplitude T_{11} as can be clearly seen from Eq. (2) and the rescattering effects is negligible. If we do not take the absorption effects into account we would obtain a much larger enhancement factor R^{11} in magnitude, $R^{11} = (1.67 - 2.02i)\times 10^{-2}$. We also estimated the rescattering amplitude $B \to {K^*}^+\rho^- \to K^0\pi^0$ in which K–exchange also contributes (in the u–channel) and found that

$$R^{00} = -(0.44+1.17i)\times 10^{-2} \ , \qquad (5)$$

and R^{11} again negligible after considering the absorption effects. From the results of Eqs. (4) and (5) we predict that a final state rescattering via π (and K) exchange contributes an enhancement factor of order of $O(\lambda^2)$, which confirms the results given in Ref. [2].

It was recently claimed [9] that the "charm penguin" effects which are neglected in the naive factorization approximation have strong effects on B decay amplitudes into light hadrons. These effects are of FSI nature, which, in

the present language, imply that there may exist strong final state rescattering effects in, say, $B \to D^+ D_s^- \to \pi^+ K^-$ processes. Motivated by this we in the following estimate the rescattering amplitude mediated by D^* and D^{**} exchanges. Based upon an estimation on the absorptive part of the rescattering amplitude, it was pointed out that [1] the D^* and D^{**} Reggeons have a very small intercept parameter ($\alpha^0 \simeq -1$) and therefore their effects are much smaller than the ρ exchange contributions. [2] To give an order of magnitude estimate we use SU(4) approximation to fix the corresponding coupling constants. We perform the dispersive integral in equation (1) from the physical threshold $s^{th}_{phys} = (m_{D^+} + m_{D_S^-})^2$ to ∞ and obtain,

$$R \equiv \frac{A(B \to D^+ D_s^- \to \pi^+ K^-)}{A(B \to D^+ D_s^-)} = (0.56 + 1.76i) \times 10^{-3} \ , \tag{6}$$

which is one order of magnitude smaller than a typical ρ exchange contribution ($\sim O(\lambda^2)$). The effect of large branching ratio of $\mathrm{Br}(B \to D^+ D_s^-)$ comparing with $\mathrm{Br}(B \to \pi^+ K^-)$ is greatly reduced by such a tiny rescattering effect,

$$|\frac{A(B \to D^+ D_s^- \to \pi^+ K^-)}{A(B \to \pi^+ K^-)}| = |R| \sqrt{\frac{Br(B \to D^+ D_s^-}{Br(B \to \pi^+ K^-)}} \simeq 0.1 \ , \tag{7}$$

in which we take $Br(B \to \pi^+ K^-) \simeq 2 \times 10^{-6}$ from the naive factorization estimate [9]. From the above result we see that the rescattering effects is still small, if not negligible.

To conclude we pointed out that the FSI effects due to pion exchanges are small. For the D^* (D^{**}) exchange the enhancement factor is still not large enough to explain the discrepancy between the naive factorization results and experiments. This may imply that the formalism we use in this note is inadequate to reveal the charm penguin effects provide that the naive factorization estimate is reliable. Another possibility is that one has to sum up all the intermediate states rather than $D^+ D_s^-$ only. In such a situation our method can not handle the multi-particle intermediate states. However we insist that our method is effective in explaining the $2 \to 2$ final state rescatterings mediated by normal Reggeon exchanges, since the dynamics of the final state interaction without charmed particles and the one with a single charmed particle are essentially the same and in the later case theory and experiments are found to agree with each other.

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[2] The suppression due to the small intercept here is slightly compensated by the slower t dependence of the Regge trajectory ($\alpha' \simeq 0.5$), which implies a larger s–wave component of the rescattering amplitude.

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